

ROUGH NEUTROSOPHIC SETS

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Abstract. Both neutrosophic sets theory and rough sets theory are emerging as powerful tool for managing uncertainty, indeterminate, incomplete and imprecise information. In this paper we develop an hybrid structure called *rough neutrosophic sets* and studied their properties.

Keywords: Rough set, rough neutrosophic set.

1. Introduction

In 1982, Pawlak [1] introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. After Pawlak, there has been many models built upon different aspect, i.e., universe, relations, object, operators by many scholars [2], [3], [4], [5], [6], [7]. Various notions that combine rough sets and fuzzy sets, vague set and intuitionistic fuzzy sets are introduced, such as rough fuzzy sets, fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vagues sets. The theory of

rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [8], [9]. Neutrosophic sets described by three functions: a membership function indeterminacy function and a non-membership function that are independently related. The theory of neutrosophic set have achieved great success in various areas such as medical diagnosis [10], database [11], [12], topology [13], image processing [14], [15], [16], and decision making problem [17]. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics, none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory, soft set theory [23], [24], [25], [26], [27], [28], [29], [30], [31], [32] and so on. The main objective of this study was to introduce a new hybrid intelligent structure called *rough neutrosophic sets*. The significance of introducing hybrid set structures is that the computational techniques based on any one of theses structures alone will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in Section 2. In Section 3, the concept of rough neutrosophic sets is investigated. Section 4 concludes the paper.

2. Preliminaries

In this section we present some preliminaries which will be useful to our work in the next section. For more details the reader may refer to [1], [8], [9].

Definition 2.1. [8] Let X be an universe of discourse, with a generic element in X denoted by x , the neutrosophic (NS) set is an object having the form

$$A = \{ \langle x : \mu_A(x), \nu_A(x), \omega_A(x) \rangle, x \in X \},$$

where the functions $\mu, \nu, \omega : X \rightarrow]^{-}0, 1^{+}[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition

$$(1) \quad ^{-}0 \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^{+}.$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So, instead of $]^{-}0, 1^{+}[$ we need to take the interval $[0, 1]$ for technical applications, because $]^{-}0, 1^{+}[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x) \rangle \mid x \in X\} \text{ and} \\ B = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x) \rangle \mid x \in X\},$$

the relations are defined as follows:

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$, $\omega_A(x) \geq \omega_B(x)$,
- (ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$,
- (iii) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)), \max(\omega_A(x), \omega_B(x)) \rangle \mid x \in X\}$,
- (iv) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)), \min(\omega_A(x), \omega_B(x)) \rangle \mid x \in X\}$,
- (v) $A^C = \{\langle x, \omega_A(x), 1 - \nu_A(x), \mu_A(x) \rangle \mid x \in X\}$
- (vi) $0_n = (0, 1, 1)$ and $1_n = (1, 0, 0)$.

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in $[0, 1]$ and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U , such that,

$$A = \{\langle x_1, (0.3, 0.5, 0.6) \rangle, \langle x_2, (0.3, 0.2, 0.3) \rangle, \langle x_3, (0.3, 0.5, 0.6) \rangle\},$$

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.6 etc.

Definition 2.3. [1] Let U be any non-empty set. Suppose R is an equivalence relation over U . For any non-null subset X of U , the sets

$$A_1(x) = \{x : [x]_R \subseteq X\} \text{ and } A_2(x) = \{x : [x]_R \cap X \neq \emptyset\}$$

are called the lower approximation and upper approximation, respectively of X , where the pair $S = (U, R)$ is called an approximation space. This equivalent relation R is called indiscernibility relation.

The pair $A(X) = (A_1(x), A_2(x))$ is called the rough set of X in S . Here $[x]_R$ denotes the equivalence class of R containing x .

Definition 2.4. [1] Let $A = (A_1, A_2)$ and $B = (B_1, B_2)$ be two rough sets in the approximation space $S = (U, R)$. Then,

$$\begin{aligned} A \cup B &= (A_1 \cup B_1, A_2 \cup B_2), \\ A \cap B &= (A_1 \cap B_1, A_2 \cap B_2), \\ A \subseteq B &\text{ if } A \cap B = A, \\ \sim A &= \{U - A_2, U - A_1\}. \end{aligned}$$

3. Rough neutrosophic sets

In this section we introduce the notion of rough neutrosophic sets by combining both rough sets and neutrosophic sets. and some operations viz. union, intersection, inclusion and equalities over them. Rough neutrosophic set are the generalization of rough fuzzy sets [2] and rough intuitionistic fuzzy sets [22].

Definition 3.1. Let U be a non-null set and R be an equivalence relation on U . Let F be neutrosophic set in U with the membership function μ_F , indeterminacy function ν_F and non-membership function ω_F . The lower and the upper approximations of F in the approximation (U, R) denoted by $\underline{N}(F)$ and $\overline{N}(F)$ are respectively defined as follows:

$$\begin{aligned} \underline{N}(F) &= \{ \langle x, \mu_{\underline{N}(F)}(x), \nu_{\underline{N}(F)}(x), \omega_{\underline{N}(F)}(x) \rangle \mid y \in [x]_R, x \in U \}, \\ \overline{N}(F) &= \{ \langle x, \mu_{\overline{N}(F)}(x), \nu_{\overline{N}(F)}(x), \omega_{\overline{N}(F)}(x) \rangle \mid y \in [x]_R, x \in U \}, \end{aligned}$$

where:

$$\begin{aligned} \mu_{\underline{N}(F)}(x) &= \bigwedge_{y \in [x]_R} \mu_F(y), \quad \nu_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y), \quad \omega_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y), \\ \mu_{\overline{N}(F)}(x) &= \bigvee_{y \in [x]_R} \mu_F(y), \quad \nu_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \nu_F(y), \quad \omega_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \omega_F(y). \end{aligned}$$

So

$$0 \leq \mu_{\overline{N}(F)}(x) + \nu_{\overline{N}(F)}(x) + \omega_{\overline{N}(F)}(x) \leq 3$$

and

$$\mu_{\underline{N}(F)}(x) + \nu_{\underline{N}(F)}(x) + \omega_{\underline{N}(F)}(x) \leq 3,$$

where " \vee " and " \wedge " mean "max" and "min" operators respectively, $\mu_F(x)$, $\nu_F(y)$ and $\omega_F(y)$ are the membership, indeterminacy and non-membership of y with respect to F . It is easy to see that $\overline{N}(F)$ and $\underline{N}(F)$ are two neutrosophic sets in U , thus the NS mappings $\overline{N}, \underline{N} : N(U) \rightarrow N(U)$ are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair $(\underline{N}(F), \overline{N}(F))$ is called the rough neutrosophic set in (U, R) .

From the above definition, we can see that $\underline{N}(F)$ and $\overline{N}(F)$ have constant membership on the equivalence classes of U , if $\underline{N}(F) = \overline{N}(F)$; i.e.,

$$\mu_{\underline{N}(F)} = \mu_{\overline{N}(F)},$$

$$\nu_{\underline{N}(F)} = \nu_{\overline{N}(F)},$$

$$\omega_{\underline{N}(F)} = \omega_{\overline{N}(F)}.$$

For any $x \in U$, we call F a definable neutrosophic set in the approximation (U, R) . It is easily to be proved that Zero O_N neutrosophic set and unit neutrosophic sets 1_N are definable neutrosophic sets. Let us consider a simple example in the following.

Example 3.2. Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

$$U/R = \{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{p_5\}, \{p_7, p_8\}\}.$$

Let

$$N(F) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.3, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2)), \\ (p_7, (0.1, 0.3, 0.5))\}$$

be a neutrosophic set of U . By Definition 3.1, we obtain:

$$\begin{aligned} \underline{N}(F) &= \{(p_1, (0.2, 0.5, 0.4)), (p_4, (0.2, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2))\}; \\ \overline{N}(F) &= \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2)), \\ &\quad (p_7, (0.1, 0.3, 0.5)), (p_8, (0.1, 0.3, 0.5))\}. \end{aligned}$$

For another neutrosophic sets

$$N(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}.$$

The lower approximation and upper approximation of $N(G)$ are calculated as

$$\begin{aligned} \underline{N}(G) &= \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}; \\ \overline{N}(G) &= \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}. \end{aligned}$$

Obviously $\underline{N}(G) = \overline{N}(G)$ is a definable neutrosophic set in the approximation space (U, R) .

Definition 3.3. If $N(F) = (\underline{N}(F), \overline{N}(F))$ is a rough neutrosophic set in (U, R) , the rough complement of $N(F)$ is the rough neutrosophic set denoted $\sim N(F) = (\underline{N}(F)^c, \overline{N}(F)^c)$, where $\underline{N}(F)^c, \overline{N}(F)^c$ are the complements of neutrosophic sets $\underline{N}(F)$ and $\overline{N}(F)$, respectively,

$$\underline{N}(F)^c = \{ \langle x, \omega_{\underline{N}(F)}, 1 - \nu_{\underline{N}(F)}(x), \mu_{\underline{N}(F)}(x) \rangle \mid x \in U \},$$

and

$$\overline{N}(F)^c = \{ \langle x, \omega_{\overline{N}(F)}, 1 - \nu_{\overline{N}(F)}(x), \mu_{\overline{N}(F)}(x) \rangle \mid x \in U \}.$$

Definition 3.4. If $N(F_1)$ and $N(F_2)$ are two rough neutrosophic set of the neutrosophic sets F_1 and F_2 respectively in U , then we define the following:

- (i) $N(F_1) = N(F_2)$ iff $\underline{N}(F_1) = \underline{N}(F_2)$ and $\overline{N}(F_1) = \overline{N}(F_2)$.
- (ii) $N(F_1) \subseteq N(F_2)$ iff $\underline{N}(F_1) \subseteq \underline{N}(F_2)$ and $\overline{N}(F_1) \subseteq \overline{N}(F_2)$.
- (iii) $N(F_1) \cup N(F_2) = \langle \underline{N}(F_1) \cup \underline{N}(F_2), \overline{N}(F_1) \cup \overline{N}(F_2) \rangle$.
- (iv) $N(F_1) \cap N(F_2) = \langle \underline{N}(F_1) \cap \underline{N}(F_2), \overline{N}(F_1) \cap \overline{N}(F_2) \rangle$.
- (v) $N(F_1) + N(F_2) = \langle \underline{N}(F_1) + \underline{N}(F_2), \overline{N}(F_1) + \overline{N}(F_2) \rangle$.
- (vi) $N(F_1) \cdot N(F_2) = \langle \underline{N}(F_1) \cdot \underline{N}(F_2), \overline{N}(F_1) \cdot \overline{N}(F_2) \rangle$.

If N, M, L are rough neutrosophic set in (U, R) , then the results in the following proposition are straightforward from definitions.

Proposition 3.5.

- (i) $\sim N(\sim N) = N$
- (ii) $N \cup M = M \cup N, N \cap M = M \cap N$
- (iii) $(N \cup M) \cup L = N \cup (M \cup L)$ and $(N \cap M) \cap L = N \cap (M \cap L)$
- (iv) $(N \cup M) \cap L = (N \cap M) \cap (N \cup L)$ and $(N \cap M) \cup L = (N \cap M) \cup (N \cap L)$.

De Morgan 's Laws are satisfied for neutrosophic sets:

Proposition 3.6.

- (i) $\sim (N(F_1) \cup N(F_2)) = (\sim N(F_1)) \cap (\sim N(F_2))$
- (ii) $\sim (N(F_1) \cap N(F_2)) = (\sim N(F_1)) \cup (\sim N(F_2))$.

Proof. (i) $(N(F_1) \cup N(F_2)) = \sim (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\sim \{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \sim \{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}^c, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}^c) = (\sim \{\underline{N}(F_1) \cap \underline{N}(F_2)\}, \sim \{\overline{N}(F_1) \cap \overline{N}(F_2)\}) = (\sim N(F_1)) \cap (\sim N(F_2)).$

(ii) Similar to the proof of (i).

Proposition 3.7. *If F_1 and F_2 are two neutrosophic sets in U such that $F_1 \subseteq F_2$, then $N(F_1) \subseteq N(F_2)$*

- (i) $N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2),$
- (ii) $N(F_1 \cap F_2) \subseteq N(F_1) \cap N(F_2).$

Proof.

$$\begin{aligned}
 \mu_{\underline{N}(F_1 \cup F_2)}(x) &= \inf\{\mu_{F_1 \cup F_2}(x) \mid x \in X_i\} \\
 &= \inf(\max\{\mu_{F_1}(x), \mu_{F_2}(x) \mid x \in X_i\}) \\
 &\geq \max\{\inf\{\mu_{F_1}(x) \mid x \in X_i\}, \inf\{\mu_{F_2}(x) \mid x \in X_i\}\} \\
 &= \max\{\mu_{\underline{N}(F_1)}(x_i), \mu_{\underline{N}(F_2)}(x_i)\} \\
 &= \mu_{\underline{N}(F_1) \cup \underline{N}(F_2)}(x_i).
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 \nu_{\underline{N}(F_1 \cup F_2)}(x_i) &\leq (\nu_{\underline{N}(F_1)} \cup \nu_{\underline{N}(F_2)})(x_i) \\
 \omega_{\underline{N}(F_1 \cup F_2)}(x_i) &\leq (\omega_{\underline{N}(F_1)} \cup \omega_{\underline{N}(F_2)})(x_i)
 \end{aligned}$$

Thus,

$$\underline{N}(F_1 \cup F_2) \supseteq \underline{N}(F_1) \cup \underline{N}(F_2).$$

We can also see that

$$\overline{N}(F_1 \cup F_2) = \overline{N}(F_1) \cup \overline{N}(F_2).$$

Hence,

$$N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2).$$

(ii) The proof of (ii) is similar to the proof of (i).

Proposition 3.8.

- (i) $N(F) = \sim \overline{N}(\sim F)$
- (ii) $\overline{N}(F) = \sim \underline{N}(\sim F)$
- (iii) $\underline{N}(F) \supseteq \overline{N}(F)$.

Proof. According to Definition 3.1, we can obtain

$$\begin{aligned}
 \text{(i)} \quad F &= \{\langle x, \mu_F(x), \nu_F(x), \omega_F(x) \rangle \mid x \in X\} \\
 \sim F &= \{\langle x, \omega_F(x), 1 - \nu_F(x), \mu_F(x) \rangle \mid x \in X\} \\
 \overline{N}(\sim F) &= \left\{ \left\langle x, \omega_{\overline{N}(\sim F)}(x), 1 - \nu_{\overline{N}(\sim F)}(x), \mu_{\overline{N}(\sim F)}(x) \right\rangle \mid y \in [x]_R, x \in U \right\} \\
 \sim \overline{N}(\sim F) &= \left\{ \left\langle x, \mu_{\overline{N}(\sim F)}(x), 1 - (1 - \nu_{\overline{N}(\sim F)}(x)), \omega_{\overline{N}(\sim F)}(x) \right\rangle \mid y \in [x]_R, x \in U \right\} \\
 &= \left\{ \left\langle x, \mu_{\overline{N}(\sim F)}(x), \nu_{\overline{N}(\sim F)}(x), \omega_{\overline{N}(\sim F)}(x) \right\rangle \mid y \in [x]_R, x \in U \right\}
 \end{aligned}$$

where

$$\mu_{\overline{N}(\sim F)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y), \quad \nu_{\overline{N}(\sim F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y), \quad \omega_{\overline{N}(\sim F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y).$$

Hence $\underline{N}(F) = \sim \overline{N}(\sim F)$.

(ii) The proof is similar to the proof of (i).

(iii) For any $y \in \underline{N}(F)$, we can have

$$\mu_{\underline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y) \leq \bigvee_{y \in [x]_R} \mu_F(y), \quad \nu_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y) \geq \bigwedge_{y \in [x]_R} \nu_F(y)$$

$$\text{and } \omega_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y) \geq \bigwedge_{y \in [x]_R} \omega_F(y).$$

Hence $\underline{N}(F) \subseteq \overline{N}(F)$.

4. Conclusion

In this paper we have defined the notion of rough neutrosophic sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and neutrosophic theory. While neutrosophic set theory is mainly concerned with, indeterminate and inconsistent information, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently, by the way they are defined, it is clear that rough neutrosophic sets can be utilized for dealing with both of indeterminacy and incompleteness.

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